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Quark mass matrices and observable quantities

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ABSTRACT

A nearly historical account of quark mass matrix models is given, and the structure of quark mass matrices in the Standard Model is studied. For a minimal parameter basis suggested earlier, where M_u is diagonal and M_{d11}, M_{d13}, M_{d31} are zero, the dependence of mass matrices on the CP violating phase δ of V_{CKM} is reported: all parameters are almost independent, except M_{d22} and M_{d23}, and the equality $|M_{d22}| = M_{d23}$ is obtained for a value of δ very close to the value which is favoured by experiments. Moreover, on this basis, $M_{d12} \simeq M_{d21}$ and $M_{d33} \simeq 2M_{d32}$. Some comments on mass matrices in left-right symmetric models are added.

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I. INTRODUCTION

Understanding the pattern of fermion masses and mixings is a key problem of particle physics. In this paper we study quark masses and mixings in the framework of the standard $SU(3)\times SU(2)\times U(1)$ gauge theory. We discuss some possible forms of quark mass matrices at the scale $\mu = M_Z$, and what they suggest. It is well known that the Standard Model (SM) does not predict fermion parameters, and looking at the structure of mass matrices we can yield hints towards a more fundamental theory.

The part of the SM Lagrangian we are interested in is formed by the quark mass and charged weak current terms

$$L_M = \overline{u}_L M_u u_R + \overline{d}_L M_d d_R + g\overline{u}_L d_L W. \tag{1}$$

When we diagonalize the mass matrices M_u and M_d we get (renaming the quark fields)

$$L_D = \overline{u}_L D_u u_R + \overline{d}_L D_d d_R + g\overline{u}_L V_{CKM} d_L W \tag{2}$$

where the mixing matrix V_{CKM} [1] contains three angles and one phase. For the standard and Wolfenstein [2] parameterizations of V_{CKM} we refer to [3]. For two generations the mixing matrix becomes a rotation

$$V_C = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \tag{3}$$

where θ_C is the Cabibbo angle. We take the numerical values of quark masses (at $\mu = M_Z$) from ref. [4] and of the mixing angles from ref. [3]; we set $\lambda = 0.22$, with $\sin\theta_C \simeq \lambda$. There is a clear hierarchy of quark masses: $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$; $m_b \ll m_t$. Moreover, V_{CKM} is near the identity and $V_{ub} \ll V_{cb} \ll V_{us}$.

L_D is expressed by means of the ten observable quantities. On the contrary, M_u and M_d, appearing in L_M, contain, in general, eighteen real parameters each. As a matter of fact, in the SM, it is possible to perform, without physical consequences, the following unitary transformations on the quark fields

$$u_L \to U u_L,\ d_L \to U d_L; \tag{4}$$

$$u_R \to V_u u_R,\ d_R \to V_d d_R. \tag{5}$$

In particular we can absorb some non physical phases in M_u and M_d by means of the transformations

$$u_L \to \mathrm{diag}(1, e^{i\varphi_1}, e^{i\varphi_2}) u_L,\ d_L \to \mathrm{diag}(1, e^{i\varphi_1}, e^{i\varphi_2}) d_L; \tag{6}$$

$$u_R \to \mathrm{diag}(e^{i\varphi_3}, e^{i\varphi_4}, e^{i\varphi_5}) u_R,\ d_R \to \mathrm{diag}(e^{i\varphi_6}, e^{i\varphi_7}, e^{i\varphi_8}) d_R. \tag{7}$$

In this way a maximum of five phases in M_1 and of three in M_2 can be absorbed. Using transformations (4),(5) (and (6),(7)) one can also get zeros in mass matrices or relations between elements, and reduce the number of independent parameters to ten. When such a basis is achieved we talk about a minimal parameter (m.p.) basis. For example, one can yield hermitian mass matrices [5]. In fact, by a polar decomposition theorem, $M = HX$, where H is hermitian and X is unitary, and by means of different V_u, V_d, hermitian mass matrices can be obtained. In particular, it is always possible to choose one matrix diagonal and the other hermitian [6]. In fact, M_1 can be diagonalized by a biunitary transformation $U^+ M_1 V_1$, and M_2 becomes hermitian by the product $U^+ M_2 V_2$. With hermitian matrices one can then take $U = V_u = V_d$. Nevertheless the freedom in eqns.(4),(5) was not much used to get zeros till the paper [7]. Hence, our paper is divided in two main parts: in section II we review hermitian quark mass matrix models and in section III we study mass matrices on some interesting weak bases. One of such bases [8] leads to a predictive reduction of independent parameters. In the final section we match the two approaches and try some conclusions in the context of a left-right symmetric gauge group.

II. REVIEW

The starting point of quark mass matrix models can be considered to rise from the following relation between the Cabibbo angle and a meson mass ratio [9]:

$$\sin^2\theta_C \simeq \frac{m_\pi^2}{2m_K^2}. \tag{8}$$

In fact, the r.h.s. of eqn.(8) is related to a quark mass ratio [10],

$$\frac{m_\pi^2}{2m_K^2} \simeq \frac{m_d}{m_s}, \tag{9}$$

and then

$$\sin\theta_C \simeq \sqrt{\frac{m_d}{m_s}}. \tag{10}$$

Weinberg [10] obtained such a relation from a particular structure of quark mass matrices. Let us consider a real symmetric matrix

$$M = \begin{pmatrix} A & B \\ B & D \end{pmatrix} \tag{11}$$

which can be diagonalized by an orthogonal transformation $O^T M O = D$ with a mixing angle θ given by

$$\tan 2\theta = \frac{2B}{D-A}. \tag{12}$$

In the limit $A \to 0$ and if $|m_1| \ll |m_2|$ we have also the approximate eigenvalues

$$m_2 \simeq D, \quad m_1 \simeq -\frac{B^2}{D} \tag{13}$$

and the relation

$$\sin\theta \simeq \frac{B}{D} \simeq \sqrt{-\frac{m_1}{m_2}}. \tag{14}$$

Then we have $B \simeq \sqrt{-m_1 m_2}$ and

$$M \simeq \begin{pmatrix} 0 & \sqrt{-m_1 m_2} \\ \sqrt{-m_1 m_2} & m_2 \end{pmatrix}. \tag{15}$$

Now, m_1 (or m_2) is negative, and if $M = M_d$ we obtain

$$D_d = \begin{pmatrix} -m_d & 0 \\ 0 & m_s \end{pmatrix}, \tag{16}$$

$$\sin\theta \simeq \sqrt{\frac{m_d}{m_s}}. \tag{17}$$

Actually, in Weinberg's paper, starting from an arbitrary real matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{18}$$

one first gets $A = 0$ by an orthogonal transformation on right-handed fields, and then assumes $B = C$. In any case, for the first two generations of quarks, if we set $M_u = D_u$ and

$$M_d = \begin{pmatrix} 0 & B \\ B & D \end{pmatrix}, \tag{19}$$

we have $B \ll D$ and formula (10). We can now express the quark mass matrices in terms of the quark masses by

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix}, \; M_u = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}. \tag{20}$$

In view of the fact that the elementary particle theory is probably more symmetric than the SM, we are led to consider a similar structure of both mass matrices. This was achieved in the famous Fritzsch model [11]. For two generations

$$M_d = \begin{pmatrix} 0 & B \\ B & D \end{pmatrix}, \; M_u = \begin{pmatrix} 0 & B' \\ B' & D' \end{pmatrix} \tag{21}$$

imply $B \ll D$, $B' \ll D'$ and

$$\sin\theta_C \simeq \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \approx \sqrt{\frac{m_d}{m_s}}. \tag{22}$$

The first generation gets its mass by mixing with the heavier second generation and we have

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix}, \; M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} \\ \sqrt{m_u m_c} & m_c \end{pmatrix}. \tag{23}$$

Here one has to note that

$$M_d \simeq m_s \begin{pmatrix} 0 & \lambda \\ \lambda & 1 \end{pmatrix}, \; M_u \simeq m_c \begin{pmatrix} 0 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix}, \tag{24}$$

with the zero in position 1-1 which could be approximate: naturalness [12,13] leads to $M_{d11} \lesssim \lambda^2$, $M_{u11} \lesssim \lambda^4$, respectively. The hierarchical form of mass matrices in eqn.(24) is due to the hierarchy of quark masses, that is matrices in eqn.(24) lead to large quark mass ratios and also to small mixing. It is then important to understand how such a form can arise. In ref. [14] a qualitative answer to this question is given by means of a broken continuous abelian symmetry beyond the SM.

For three generations of quarks the Fritzsch model is given by

$$M_d = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}, \; M_u = \begin{pmatrix} 0 & A' & 0 \\ A'^* & 0 & B' \\ 0 & B'^* & C' \end{pmatrix}, \tag{25}$$

and with $A \ll B \ll C$, $A' \ll B' \ll C'$ one yields the relations

$$V_{us} \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\sigma} \sqrt{\frac{m_u}{m_c}} \right|, \tag{26}$$

$$V_{cb} \simeq \left| \sqrt{\frac{m_s}{m_b}} - e^{i\rho} \sqrt{\frac{m_c}{m_t}} \right|, \tag{27}$$

but setting $m_t = 180$ GeV we obtain a too large V_{cb} (from 0.10 to 0.25, using the central values of quark masses). The value of σ must be close to $\pm\pi/2$, because eqn.(10) is already well satisfied. In this model the two lighter generations get mass by direct and indirect mixing with the heavier third generation. An alternative model, with a different structure of M_d, is due to Georgi and Jarlskog [15] (but see also ref. [16]),

$$M_d = \begin{pmatrix} 0 & A & 0 \\ A^* & B & 0 \\ 0 & 0 & C \end{pmatrix}, \; M_u = \begin{pmatrix} 0 & A' & 0 \\ A' & 0 & B' \\ 0 & B' & C' \end{pmatrix}, \tag{28}$$

which gives a relation similar to (26) and the new relation

$$V_{cb} \simeq \sqrt{\frac{m_c}{m_t}}. \tag{29}$$

Thus, $V_{cb} = 0.061 \pm 0.005$ is of the right order but again too large. We can write

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \; M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & 0 & \sqrt{m_c m_t} \\ 0 & \sqrt{m_c m_t} & m_t \end{pmatrix}. \tag{30}$$

Actually, this model has been studied in SU(5) [15] and SO(10) [17] and also in supersymmetric versions [18]. There the charged lepton mass matrix is related to M_d and, in the SO(10) model, one also has predictions on the neutrino sector.

The Georgi-Jarlskog model can be seen as a Fritzsch model for two generations for M_d plus a Fritzsch model for three generations for M_u. Other modifications of the Fritzsch model consist in taking elements 1-3 or 2-2 different from zero. Neither the modification of the element 1-3 [19] nor the element 2-2 in M_u [20], can give the correct V_{cb}. However this can be obtained with the element 2-2 in M_d and then with the same structure in both M_d and M_u [21]

$$M_d = \begin{pmatrix} 0 & A & 0 \\ A & D & B \\ 0 & B & C \end{pmatrix}, \; M_u = \begin{pmatrix} 0 & A' & 0 \\ A'^* & D' & B' \\ 0 & B'^* & C' \end{pmatrix}, \tag{31}$$

with a relation similar to (26), and

$$\frac{V_{ub}}{V_{cb}} \simeq \sqrt{\frac{m_u}{m_c}}. \tag{32}$$

In this case only the first generation gets mass by just mixing. Models that lead to matrices similar to (31) have been studied by several authors [22] (the phase in B' can be shifted to A, or supposed to be zero). A flavor permutation symmetry breaking [23] is often used. In refs. [24] and [25] the mass matrices are written as

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b \end{pmatrix}, \; M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t \end{pmatrix}, \tag{33}$$

yielding a new successful relation

$$V_{cb} \simeq \sqrt{\frac{m_d}{m_b}}. \tag{34}$$

As hermitian matrices, such models have four texture zeros (two zeros in symmetric positions are counted as one texture zero). A systematic analysis of five texture zero matrices, in the $SO(10)$ model, has been carried out by Ramond, Roberts, and Ross (RRR) [26]. They found five solutions which were consistent with low energy data (all matrices have an approximate hierarchical expression in terms of powers of λ; for a matching of RRR analysis with the idea of naturalness see [13]). Let us consider their solution 3:

$$M_d = \begin{pmatrix} 0 & A & 0 \\ A^* & B & C \\ 0 & C & D \end{pmatrix}, \; M_u = \begin{pmatrix} 0 & 0 & C' \\ 0 & B' & 0 \\ C' & 0 & D' \end{pmatrix}. \tag{35}$$

The phase in A can be shifted to C' and in ref. [27] and ref. [24] (see also [28]) these matrices are written as

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b \end{pmatrix}, \; M_u \simeq \begin{pmatrix} 0 & 0 & \sqrt{m_u m_t} \\ 0 & m_c & 0 \\ \sqrt{m_u m_t} & 0 & m_t \end{pmatrix}, \tag{36}$$

leading to the mixings

$$V_{us} \simeq \sqrt{\frac{m_d}{m_s}}, \tag{37}$$

$$V_{cb} \simeq \frac{m_s}{m_b}, \tag{38}$$

coming from the down sector, while the mixing

$$V_{ub} \simeq \sqrt{\frac{m_u}{m_t}} \tag{39}$$

comes from the up sector. Also consider the hierarchical expressions for eqns.(31) and (35):

$$M_d \simeq m_b \begin{pmatrix} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \; M_u \simeq m_t \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ 0 & \lambda^4 & 1 \end{pmatrix}; \tag{40}$$

$$M_d \simeq m_b \begin{pmatrix} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \; M_u \simeq m_t \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \lambda^4 & 0 \\ \lambda^4 & 0 & 1 \end{pmatrix}. \tag{41}$$

Again the appearance of zeros and the hierarchical structure of mass matrices point towards some broken horizontal symmetry (for example $U(1)_H$ [29]) with a breaking depending on the small parameter λ. The texture zeros should be zero only up to the order that does not change masses and mixings. If the symmetry is exact, only the third generation would be massive and the mixings vanish. Instead the terms that break the symmetry, gradually fill in the mass matrices with powers of λ, generating the hierarchy of masses and mixings. Hence a broken symmetry can explain both the approximate zeros in mass matrices and the hierarchy of non vanishing elements. Such a symmetry could also work in the context of a unified or string theory.

Another approach to matrix models of fermion masses and mixings is the above mentioned permutation symmetry breaking and a suggestive variation of it [30], which is called USY (Universal Strength of Yukawa couplings, that is phase mass matrices).

To conclude this section we stress that two forms of quark mass matrices which agree with numerical values of quark masses and mixing, namely eqn.(31) and eqn.(35), are reported. The first has the same zeros in M_u and M_d and, with a real B' (only one phase is necessary for CP violation), contains nine real parameters. The second has a non parallel structure of zeros and contains eight real parameters. Of course, other forms may be considered [31]. In the following section we use from the beginning the m.p. basis approach to discover relations and properties of mass matrices.

III. WEAK BASES

In ref. [7] it was shown that, in the SM (with less than five generations), with a choice of the weak basis, one can always obtain the zeros of the Fritzsch model. This is called the NNI (nearest neighbor interaction) basis. Using transformations (4),(5) it is always possible to go to the NNI basis. Then, using the rephasing of quark fields, one matrix has no phases and the other two phases, twelve real parameters in all. As the observables quantities are ten, it is important to go to a basis where, after rephasing, ten parameters are left (although not minimal the non hermitian NNI basis is interesting, see ref. [32]). Using transformations (4),(5) it is in fact possible to get several interesting m.p. bases.

Let us begin with the case of only two generations. In such a case we have five observables: four masses and one mixing angle. As shown in the introduction, one can always diagonalize M_1 and then use V_2 to obtain some special form of M_2. This is true for an arbitrary number of generations, allowing to shift the mixings to a single mass matrix. Moreover, the diagonal matrix is real and non negative. For example, if we set $M_u = D_u$ and M_d hermitian (and also real), then we get (using the central values of quark masses, in GeV; $V \equiv V_{CKM}$)

$$M_d = VD_dV^+ = \begin{pmatrix} 0.009 & 0.019 \\ 0.019 & 0.088 \end{pmatrix} \simeq m_s \begin{pmatrix} \lambda^2 & \lambda \\ \lambda & 1 \end{pmatrix} . \tag{42}$$

Starting from $M_u = D_u$ and M_d real and symmetric, we can get a zero in M_{d11} [10] or M_{d12} [33], by means of a rotation of right-handed fields. Then $M_d M_d^+ = V D_d^2 V^+$ gives

$$M_d = \begin{pmatrix} 0 & a \\ b & c \end{pmatrix} = \begin{pmatrix} 0 & 0.021 \\ 0.023 & 0.088 \end{pmatrix}, \tag{43}$$

with $M_{d12} \simeq M_{d21}$, and we recover the Weinberg model of section II, or

$$M_d = \begin{pmatrix} a & 0 \\ c & b \end{pmatrix} = \begin{pmatrix} 0.021 & 0 \\ 0.088 & 0.023 \end{pmatrix}, \tag{44}$$

with $M_{d11} \simeq M_{d22}$. Therefore, on these bases, the numerical values of quark masses and mixing point towards a reduction of independent parameters and to the relation (10).

For three generations again one can choose $M_u = D_u$ and M_d hermitian [34,4], to yield (in the diagonal bases the non diagonal matrix has one physical phase but three of them preserve arbitrary representation of V_{CKM})

$$M_d = \begin{pmatrix} 0.009 & 0.019 & 0.010e^{i\varphi} \\ 0.019 & 0.093 & 0.113 \\ 0.010e^{-i\varphi} & 0.113 & 2.995 \end{pmatrix} = m_b \begin{pmatrix} 0.003 & 0.006 & 0.003e^{i\varphi} \\ 0.006 & 0.031 & 0.038 \\ 0.003e^{-i\varphi} & 0.038 & 0.998 \end{pmatrix}. \tag{45}$$

It is also possible to get a m.p. basis inside the NNI basis [35]. Let us set $H = MM^+$. From arbitrary mass matrices first move to a basis with $H_u = D_u^2$ [36]. Then, by means of a unitary matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha}c & e^{i\beta}s \\ 0 & -e^{i\gamma}s & e^{i\delta}c \end{pmatrix}, \ \alpha + \gamma - \beta = \delta, \tag{46}$$

one can get $H'_u = U H_u U^+$, with $H'_{u12} = H'_{u13} = 0$, and $H'_d = U H_d U^+$, with $H'_{d12} = 0$, that is [7,35] M_d in the NNI form, and M_u in the NNI form and with the element 3-2 equal to zero. Rephasing the quark fields only one phase remains in M_u and we have a m.p. basis.

Now we turn to m.p. bases with $M_u = D_u$ and M_d not hermitian but containing three zeros. In such cases equation $M_d M_d^+ = V D_d^2 V^+$ enable us to calculate M_d. The case $M_d = D_d$ seems less interesting. The first of such diagonal bases to be studied [37] has zeros in positions 1-1, 2-2 and 3-1. This basis however does not seem to lead to useful relations. The lower triangular form [38] gives the numerical values

$$M_d = \begin{pmatrix} 0.023 & 0 & 0 \\ 0.104 & 0.106 e^{i\varphi} & 0 \\ 1.213 & 2.687 & 0.541 \end{pmatrix} \tag{47}$$

showing $M_{d21} \simeq M_{d22}$ and $M_{d32} \simeq 2M_{d31}$. In ref. [8] M_d has the zeros in symmetric positions [6] and, with the updated values of mixings,

$$M_d = \begin{pmatrix} 0 & 0.023 & 0 \\ 0.021 & 0.104 e^{i\varphi} & 0.104 \\ 0 & 1.213 & 2.741 \end{pmatrix}. \tag{48}$$

This suggests to take

$$M_d \simeq \begin{pmatrix} 0 & a & 0 \\ a & b e^{i\varphi} & b \\ 0 & c & 2c \end{pmatrix} \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s e^{i\varphi} & m_s \\ 0 & m_b/\sqrt{5} & 2m_b/\sqrt{5} \end{pmatrix}, \tag{49}$$

with the same 1-2 submatrix as in eqn.(20), yielding the relations (10),

$$V_{cb} \simeq \frac{3}{\sqrt{5}} \frac{m_s}{m_b}, \tag{50}$$

$$\frac{V_{ub}}{V_{cb}} \simeq \frac{1}{3} V_{us}, \tag{51}$$

and the hierarchical expression

$$M_d \simeq m_b \begin{pmatrix} 0 & \lambda^3/\sqrt{2} & 0 \\ \lambda^3/\sqrt{2} & \lambda^2/\sqrt{2} & \lambda^2/\sqrt{2} \\ 0 & 1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix}. \tag{52}$$

$\delta_{CP}(^o)$	30	40	50	60	70	80	90	100	110	120	130	140
$\|M_{d22}\|$	124	121	117	112	107	100	94	86	79	70	62	54
M_{d23}	78	83	88	94	100	107	113	119	124	129	132	136
$-\varphi(^o)$	32	42	50	58	65	72	78	85	91	98	106	115

Table 1

For the Jarlskog's parameter J [39], which is related to CP violation in the SM and given by the relation

$$\det[H_d, H_u] = 2B \cdot T \cdot J, \tag{53}$$

where $B = (m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)$, $T = (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)$, and $J = (-1)^{r+s}\mathrm{Im}(V_{ij}V_{kl}V_{kj}^*V_{il}^*)$ is obtained crossing out row r and column s of V_{CKM}, we have the approximate expression

$$J \simeq \frac{3}{5}\frac{m_d m_s}{m_b^2}\sin\delta_{CP}. \tag{54}$$

It can be checked that the relation between M_{d22} and M_{d23} is sensitive to variations of the CP violating phase δ_{CP} in V_{CKM}. Equality $M_{d23} = |M_{d22}|$ is obtained for $\delta_{CP} = 75^o$, which is very close to the experimentally favoured value [3,40]. For $\delta_{CP} = 126^o$, which is near the bound of the allowed region, it happens $M_{d23} = 2|M_{d22}|$. All others parameters in M_d are nearly independent from δ_{CP}. In table 1 we report the values of M_{d22}, M_{d23} (in MeV) and φ versus δ_{CP}. Note also that for $\delta_{CP} = 90^o$ it is $|M_{d22}| = m_s$.

One can worry about the relation $M_{d33} \approx 2M_{d23}$, but this depends on the value of V_{ub}, which has large uncertainties; for example, taking $V_{ub} = 0.0035$, the above relation is well satisfied. Therefore, the study of this basis, which has M_u diagonal and three zeros built in M_d, leads to three simple relations between matrix elements of M_d. With the actual hierarchical values of quark masses and mixings, these relations are related to the value of the phase δ_{CP}, to the relation (10), and to the value of V_{ub}, respectively.

This last diagonal three zero basis can be obtained from the first by a unitary transformation of the form (46) on d_R. Relabeling indices for the three diagonal bases leads to other fifteen bases, that however do not seem more interesting than the basis (48). Actually, starting from the first of such bases, other thirty-five bases with M_u diagonal and M_d with three zeros can be obtained by means of $U(2)$ transformations similar to (46) on d_R and relabeling. In the same way other seventeen bases can be obtained from the triangular basis. One of these is studied in detail in ref. [41].

As concerning hermitian non diagonal bases we refer to the paper [42], where eighteen bases without zeros on the diagonal are obtained. Starting from M_1 diagonal and M_2 hermitian, by the unitary transformation (46) one yields $UM_1U^+ = M_1'$ with texture zeros in positions 1-2 and 1-3, and $UM_2U^+ = M_2'$ with a texture zero in position 1-2 or 2-3. Then by relabeling indices other seventeen bases are obtained. Looking at the five RRR solutions, one realizes that solutions 1 [43], and 3, 4, 5, correspond to four of such eighteen bases with the element 1-1 of both mass matrices set equal to zero, and $M_{d23} \simeq 2M_{d22}$. In particular, matrices (36) have $M_{11} = 0$ and $M_{d23} \simeq M_{d22}$. On the contrary, matrices (31) cannot be obtained from a m.p. basis, because they already contain ten parameters (eight moduli and two phases), although one can always take $M_{13} = 0$ in both hermitian matrices [44].

IV. CONCLUDING REMARKS

In summary, some of the m.p. bases clearly show relations among the matrix elements and/or vanishing elements. In the SM different m.p. bases with different basis zeros lead to the same values of the ten observables. Instead, in left-right symmetric models, different zeros correspond to different values of the (more than ten) observables. Actually, in such models, one can often get one diagonal matrix, but the other has no freedom, because $V_u = V_d$, and then putting zeros corresponds to choosing ansatze. These must give the same left-handed currents but can produce different right-handed currents, which have not been observed till now, because the corresponding gauge bosons are expected to be heavy, but play an important role in unified theories, and affect proton decay [45]. For example, let us look at matrix (48). Viewed in the SM, such a matrix shows a hierarchical non symmetric structure with three simple relations between elements, while the zeros have no physical content. Instead, in a left-right model also the zeros (which in this case could be approximate) have a physical meaning, and the implications of matrix (48) are quite different, for example, from those of matrix (47). In a similar way, in left-right models, it is always possible to take a basis where both M_d and M_u have the zeros as in eqn.(31) [7], while taking one matrix diagonal, or the symmetric form (31), or the NNI form, gives different definite predictions. It is still left for the future to confirm the need for an extension of the gauge group at higher energy and, in such a case, the selection of the right fermion mass matrix forms.

In this paper we have considered quark mass matrices. Of course, a similar work can be done for charged lepton and neutrino mass matrices, although we do not know neutrino masses so well. In unified theories lepton mass matrices are related to quark mass matrices, but other analogies can exist between the quark and the lepton sector. For example, in ref. [25] all fermion mass matrices have the same texture zeros, and in ref. [46] a strict similarity in the Dirac sector is assumed; see also ref. [47] for a suggestive approach based on permutation symmetry. Neutrino masses and mixings are in the main stream of current physical research.

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